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LETTER TO THE EDITOR

Contact capacitance in the two-dimensional electron gas

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Abstract. The distributions of electric potential and carrier concentration, as well as the contact capacitance C, are calculated for the system of a two-dimensional electron gas with a bulk metal contact. The value of C is shown to have a logarithmic frequency dependence.

The investigation and device applications of transport effects in two-dimensional systems require the formation of electrical contacts with a two-dimensional electron gas (2DEG). These contacts have macroscopic dimensions exceeding all the characteristic lengths of the problem (2DEG layer thickness, screening length, mean free path etc) and, hence, can be described in the simple geometry shown in figure 1. Unlike in the usual contacts with bulk material, in a 2DEG the electric field caused by the contact potential difference is concentrated outside the charge-containing region (the $z = 0$ plane), which results in there being a number of qualitative distinguishing contact properties. We shall calculate one of the most important characteristics of a contact—its capacitance C—for different signal frequencies ω.

For a static case ($\omega = 0$), the charge and potential distribution in the system considered was studied in [1]. It was shown that the surface charge density far from the contact decreases rather slowly, $\sim x^{-1}$. As a result, the total charge and, hence, the contact capacitance diverge logarithmically. This means that in fact the capacitance is determined not by the microscopic parameters of the 2DEG but by the geometrical size of the contact itself or of the 2DEG plane. We shall see that for non-zero frequencies ω this is not the case; C is independent of the geometry, being characterized only by parameters of the 2DEG and the frequency.

To determine the contact capacitance, we must calculate the non-equilibrium carrier density $n_1(x)$ generated in a 2DEG by an AC voltage $V_1 \propto \exp(\mathrm{i}\omega t)$ applied between a metallic contact and some remote part of the 2DEG. n_1 is to be determined from the continuity equation

$$\mathrm{i}\omega n_1 = \mu n_0 \left(\frac{\pi\hbar^2}{em} \frac{\partial^2 n_1}{\partial x^2} - \frac{\partial^2 \phi(x,\, 0)}{\partial x^2} \right) \tag{1}$$

(we have used the Einstein relation for a degenerate 2DEG: $D = \pi\hbar^2 n_0 \mu / em$ where D, n_0 and μ are, respectively, the diffusion coefficient, density and mobility of the

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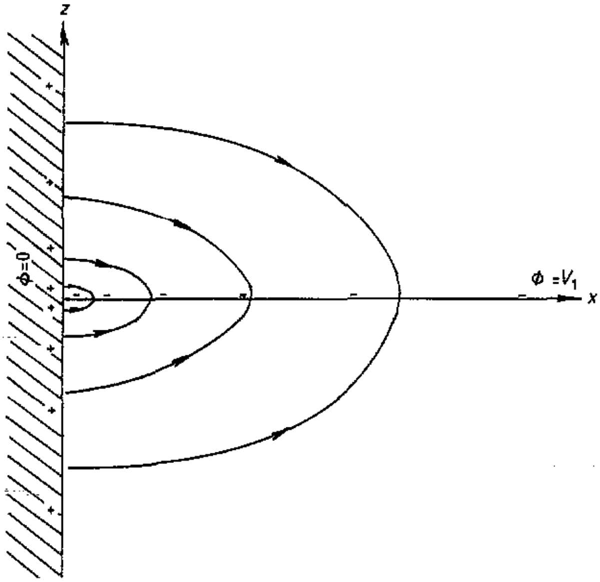

Figure 1. Schematic view of the contact with a 2DEG. The latter is considered to occupy the half-plane $z = 0$, $x > 0$ whereas the half-space $x < 0$ is the metallic contact. The electric field and charge distribution induced by a contact potential are also shown.

2DEG). The electrostatic potential $\phi(x, z)$ is found, in turn, from the Laplace equation with the boundary conditions

$$\phi(0, x) = 0 \tag{2}$$

$$\partial\phi(x, 0)/\partial z = (2\pi e/\kappa)n_1(x) \tag{3}$$

$$\phi(x \to \infty, 0) = V_1. \tag{4}$$

Here κ is the dielectric constant of a semiconductor.

The solution of the Laplace equation satisfying the conditions (2), (3) can be written in the form

$$\phi(x, z) = -\frac{4e}{\kappa}\int_0^\infty \nu(\lambda)\ \sin(\lambda x)\ \exp(-\lambda z)\ \frac{\mathrm{d}\lambda}{\lambda} \tag{5}$$

where

$$\nu(\lambda) = \int_0^\infty n_1(x)\ \sin(\lambda x)\ \mathrm{d}x$$

is the Fourier component of $n_1(x)$. After the substitution of (5) into (1) one obtains the equation for n_1 which can be solved with the help of the Fourier sine transformation.

The final result is as follows:

$$\nu(\lambda) = \lambda n_1(0)/(\lambda^2 + 2\lambda/a + \mathrm{i}/al). \tag{6}$$

This formula contains two charactersitic lengths of the problem. The first one is the effective Bohr radius $a = \kappa\hbar^2/me^2$ which plays the role of a screening length in a 2DEG [2] whereas

$$l = \pi e n_0 \mu/\kappa\omega = \pi\sigma/\kappa\omega \tag{7}$$

is the second characteristic length describing the charge relaxation processes in a 2DEG with the conductivity σ.

The unknown parameter $n_1(0)$ in (6) is determined by condition (4) which gives $n_1(0) = -\pi e V_1 m/\hbar^2$. Finally we obtain

$$n_1(x) = -\frac{2eV_1 m}{\hbar^2}\int_0^\infty \frac{\lambda\ \sin(\lambda x)\ \mathrm{d}\lambda}{\lambda^2 + 2\lambda/a + \mathrm{i}/al}. \tag{8}$$

The knowledge of the electron density distribution allows us to reach our final goal and calculate the contact capacitance per unit length:

$$C = -\frac{e}{V_1}\int n_1(x)\ \mathrm{d}x = \frac{\kappa}{\sqrt{1-\mathrm{i}a/l}}\ln\left(\frac{1+\sqrt{1-\mathrm{i}a/l}}{1-\sqrt{1-\mathrm{i}a/l}}\right). \tag{9}$$

At relatively low frequencies when $a \ll l$

$$C \simeq \kappa\left[\ln\left(\frac{4\pi m e^2\sigma}{\kappa^2\omega\hbar^2}\right) - \mathrm{i}\frac{\pi}{2}\right]. \tag{10}$$

We see that Re $C(\omega)$ diverges logarithmically in the limit $\omega \to 0$, in accordance with static calculations [1], while Im C is due to the Joule losses in a 2DEG but does not depend on σ since the width of the effective loss region, l, is also proportional to σ.

In our previous theory two simplifying assumptions were made:

(i) the equilibrium 2DEG density n_0 in (1) was assumed to be homogeneous—that is, the contact band bending was ignored;

(ii) the displacement current was neglected.

For the first assumption to be valid, the characteristic length l must exceed considerably the width of the contact space charge region $l_0 = \kappa V_0/2\pi e n_0$ (V_0 is the contact potential) so that most of the electrons moving in the AC electric field belong the the quasi-neutral region of the 2DEG. For typical 2DEG parameters this requirement is met over the whole microwave frequency range.

Assumption (ii) is *a priori* valid if the electromagnetic wavelength $c/\sqrt{\kappa\omega}$ exceeds all the characteristic lengths of the problem. The condition $c/\sqrt{\kappa\omega} \gg a$ is always fulfilled in the capacitance measurements but the condition $c/\sqrt{\kappa\omega} \gg l$ may be violated in some cases. It is interesting to note that the latter condition is, in fact, independent of frequency and equivalent to the requirement for the 2DEG conductivity σ (in CGSE units) to be much less than the light velocity c. Therefore, formula (9) for the capacitance is correct only for relatively highly resistive 2DEG layers with $\sigma/\sqrt{\kappa} \ll c/2\pi = (188\ \Omega)^{-1}$.

If $2\pi\sigma/\sqrt{\kappa} \geqslant c$, a relativistic approach is necessary even for quasi-static problems [3, 4] and instead of the Laplace equation $\Delta\phi = 0$ we must solve the Helmholtz equation $\Delta\phi + \kappa\omega^2\phi/c^2 = 0$ and a similar equation for the vector potential $\boldsymbol{A}$. This is a fairly complicated problem but the main results can be qualitatively predicted. In this case the AC current is concentrated in the region with the characteristic size $\sim c/\sqrt{\kappa\omega}$ and the losses are due to the radiation of this effective dipole. Calculations similar to (5)–(10) give

$$\mathrm{Re}\ C \simeq \kappa\ \ln\left(\frac{c}{a\sqrt{\kappa\omega}}\right) = \kappa\ \ln\left(\frac{ce^2 m}{\hbar^2\kappa^{3/2}\omega}\right) \tag{11}$$

which also diverges logarithmically at $\omega \to 0$, whereas Im $C \simeq \kappa$, as in (10).

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